Kruskal-Szekeres coordinates for Schwarzschild spacetime

$$
\left.\begin{array}{l}
\left\{\begin{array}{l}
U=\sqrt{\frac{r}{2 G M}-1} e^{\frac{r}{4 G M}} \cosh \left(\frac{t}{4 G M}\right) \\
V=\sqrt{\frac{r}{2 G M}-1} e^{\frac{r}{4 G M}} \sinh \left(\frac{t}{4 G M}\right)
\end{array}\right\} \quad \text { for } r>2 G M \\
U=\sqrt{1-\frac{r}{2 G M}} e^{\frac{r}{4 G M}} \sinh \left(\frac{t}{4 G M}\right) \\
V=\sqrt{1-\frac{r}{2 G M}} e^{\frac{r}{4 G M}} \cosh \left(\frac{t}{4 G M}\right)
\end{array}\right\} \quad \text { for } r<2 G M \text {. } \begin{aligned}
& r \\
& \Rightarrow \begin{cases}U^{2}-V^{2}=\left(\frac{r}{2 G M}-1\right) e^{\frac{r}{2 G M}} \\
V=\tanh \left(\frac{t}{4 G M}\right) U & \text { for } r>2 G M \\
V=\operatorname{coth}\left(\frac{t}{4 G M}\right) U & \text { for } r 2 G M\end{cases}
\end{aligned}
$$





